Analytical presentation of statistically estimated magnetotelluric transfer functions by a set of polynomials

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Abstract. In magnetotelluric studies time variations of the horizontal telluric and magnetic field components at the earth's surface are compared to get detailed information of the electrical conductivity structure of the earth's interior. The development of conductivity models from the data demands the thorough estimation of the transfer functions in the frequency domain between the Fourier transforms of the recorded time series. The analytical presentation of the estimated transfer functions allows an individual selection of a number of frequencies for further investigation. Larsen's presentation (Larsen, 1975, 1980) of the transfer functions by single polynomials demands a complicated calculation of confidence limits. Therefore, the transfer functions are presented here by the sum of polynomials which fulfil an orthogonality criterion. The orthogonality criterion allows a rather simple estimation of the frequency-dependent confidence limits of the transfer functions. The polynomial method is applied to a 100-day record of the magnetic and telluric field variations near Göttingen. As the telluric field is usually partially disturbed during such a long time interval, the polynomial method is extended to treat telluric time series with missing data. The comparison of the smooth polynomial transfer functions with band-averaged estimates yields a good correlation between the estimates as well as between their confidence intervals.

Key words: Confidence limits – disturbed telluric time series – Smooth polynomial MT transfer functions

Introduction

In magnetotelluric investigations time variations of the horizontal components of the telluric field, E_x and E_y, and of the three components of the magnetic field, B_x, B_y and B_z, are simultaneously recorded at the earth's surface. The comparison of E_x, E_y *or* B_z with B_x and B_y in the frequency domain may yield sophisticated models of the electric conductivity structures of the earth's interior beneath the site of interest. Thus, complex frequency-dependent transfer functions between the different Fourier-transformed field quantities have to be estimated as precisely as possible.

This paper concentrates on the estimation of transfer functions between the telluric and magnetic field variations. It first deals with Larsen's method (Larsen, 1975, 1980) which represents the transfer function by a 1-D transfer function times a power series in square-root frequency using N terms. The 1-D transfer function approximates the actual transfer functions so that the power series is approximately frequency independent and is non-dimensional. This helps in dealing with data gaps and outliers and avoids the problem that transfer functions can probably not, in general, be represented by polynomials. However, as the polynomial coefficients are statistically dependent, the full covariance matrix of their errors must be used to construct confidence limits for the resulting transfer function.

Therefore, in this paper each transfer function is presented by a sum of special polynomials which fulfil an orthogonality condition and thus offer a simpler way of calculating confidence limits.

A numerical example compares the transfer functions from the new method with band-averaged estimates. Furthermore, the influence of data gaps in the electric time series on the estimation of transfer functions is discussed.

Theory

It is assumed that there exist complete sets of N discrete and equally spaced data of the simultaneously recorded telluric and magnetic field components. Then a harmonic analysis will yield $M=N/2$ complex harmonic coefficients, e.g. $E_x(f_i)$, $i=1, \ldots, M$, where $f_i=i/T$ denotes the frequency and T is the length of the time series. A linear bivariate approach is chosen to estimate the transfer functions between the harmonic coefficients of the telluric and magnetic field

$$E(f_i)=X(f_i)\,B_x(f_i)+Y(f_i)\,B_y(f_i)+\delta E(f_i) \qquad (1)$$

where $X(f_i)$ and $Y(f_i)$ are the transfer functions which have to be estimated and $\delta E(f_i)$ is the uncorrelated residuum. $E(f_i)$ stands for either $E_x(f_i)$ or $E_y(f_i)$ as the calculations for both components are done separately but follow the same scheme. Furthermore, it is assumed that $B_x(f_i)$ and $B_y(f_i)$ are "noise-free" compared to $E(f_i)$. In the following, the argument f_i will be omitted for clarity.

Estimates of X and Y are calculated by minimizing the weighted residual power $\langle W|\delta E|^2\rangle$ by the method of least squares assuming that in the investigated period range δE is normally distributed. $W(f_i)$ is a non-negative weight function and is set to zero if there is evidence for a periodic signal at a certain frequency in the spectrum. Furthermore, W can be used to whiten the residuum δE.

There are different ways to represent the frequency dependence of X and Y:

a) In the band-average method estimates of X and Y are calculated for separate frequency bands from the normal equations yielding, e.g. for X,

$$\hat{X} = \frac{\langle EB_x^* \rangle \langle |B_y|^2 \rangle - \langle EB_y^* \rangle \langle B_y B_x^* \rangle}{\langle |B_x|^2 \rangle \langle |B_y|^2 \rangle - |\langle B_x B_y \rangle|^2} \tag{2}$$

where $\langle\ \rangle$ denotes the sum of the auto- or cross-spectra of each frequency band. However, if X is not a constant or a linear function of frequency this method might yield severely biased estimates $\hat{X}$ for wide frequency bands. On the other hand, a reduced width of the frequency bands may result in large confidence intervals.

b) Larsen (1975, 1980) describes another approach. He uses a preliminary model and then approximates the real model by the multiplication of the preliminary transfer function with an estimated power series in $\sqrt{f}$. Choosing a uniform half-space as the start model, the preliminary transfer function is proportional to $\sqrt{f}$ and thus $X(f)$ and $Y(f)$ can be expanded as follows:

$$X(f_i) = \sum_{n=0}^{N_x-1} a_n (f_i)^{\frac{n+1}{2}}, \qquad Y(f_i) = \sum_{n=0}^{N_y-1} b_n (f_i)^{\frac{n+1}{2}} \tag{3}$$

The least-squares method then serves to estimate the complex coefficients a_n and b_n. Inserting Eq. (3) into Eq. (1) will lead to the normal equations

$$\begin{aligned}
&\sum_{n=0}^{N_x-1} a_n \left\langle W|B_x|^2 f^{\frac{n+m+2}{2}} \right\rangle + \sum_{n=0}^{N_y-1} b_n \left\langle WB_x^* B_y f^{\frac{n+m+2}{2}} \right\rangle \\
&\quad = \left\langle WEB_x^* f^{\frac{m+1}{2}} \right\rangle \\
&\sum_{n=0}^{N_y-1} b_n \left\langle W|B_y|^2 f^{\frac{n+m'+2}{2}} \right\rangle + \sum_{n=0}^{N_x-1} a_n \left\langle WB_y^* B_x f^{\frac{n+m'+2}{2}} \right\rangle \\
&\quad = \left\langle WEB_y^* f^{\frac{m'+1}{2}} \right\rangle
\end{aligned} \tag{4}$$

where $\langle\ \rangle$ indicates the summation over the whole investigated frequency range and $m=0, \ldots, N_x-1$, $m'=0, \ldots, N_y-1$. Then a_n and b_n are calculated by matrix inversion. Thus, $X(f)$ and $Y(f)$ will be smooth functions of frequency and can easily be used to determine X and Y for any frequency in the analysed interval.

However, the full covariance matrix of the errors of the coefficients a_n and b_n, respectively, has to be known to calculate confidence limits for the transfer functions $X(f)$ and $Y(f)$.

c) Extending Larsen's approach, an orthogonality condition is introduced by forming the expressions

$$\begin{aligned}
X(f_i)\, B_x(f_i) &= \sum_{n=0}^{N_x-1} c_n B_{x,n}(f_i), \\
Y(f_i)\, B_y(f_i) &= \sum_{n=0}^{N_y-1} d_n B_{y,n}(f_i)
\end{aligned} \tag{5}$$

with

$$B_{x,n}(f_i) = \sqrt{f}\, B_x(f_i)\, P_n(f_i), \qquad B_{y,n}(f_i) = \sqrt{f}\, B_y(f_i)\, Q_n(f_i)$$

with P_n, Q_n being real polynomials in f_i of the order n

$$P_n(f_i) = \sum_{l=0}^{n} p_{n,l} (f_i)^l, \qquad Q_n(f_i) = \sum_{l=0}^{n} q_{n,l} (f_i)^l \tag{6}$$

such that

$$\begin{aligned}
\langle B_{x,n} B_{x,k}^*\, W \rangle &= \delta_{n,k} \langle |B_{x,n}|^2\, W \rangle, \\
\langle B_{y,n} B_{y,k}^*\, W \rangle &= \delta_{n,k} \langle |B_{y,n}|^2\, W \rangle
\end{aligned} \tag{7}$$

with $\delta_{n,k}=1$ for $n=k$ and $\delta_{n,k}=0$ for $n \neq k$.

An algorithm to calculate the polynomial coefficients $p_{n,l}$ and $q_{n,l}$ was developed by Forsythe (1957) and is described in Appendix A.

The transfer functions X and Y are now represented by the sum of the polynomials P_n and Q_n

$$X(f_i) = \sqrt{f_i} \sum_{n=0}^{N_x-1} c_n P_n(f_i), \qquad Y(f_i) = \sqrt{f_i} \sum_{n=0}^{N_y-1} d_n Q_n(f_i) \tag{8}$$

Similarly to Eq. (4), the normal equations are

$$\begin{aligned}
&\sum_{n=0}^{N_x-1} c_n \langle W|B_x|^2 P_n P_m f \rangle + \sum_{n=0}^{N_y-1} d_n \langle WB_x^* B_y Q_n P_m f \rangle \\
&\quad = \langle WB_x^* E P_m \sqrt{f} \rangle \\
&\sum_{n=0}^{N_y-1} d_n \langle W|B_y|^2 Q_n Q_{m'} f \rangle + \sum_{n=0}^{N_x-1} c_n \langle WB_y^* B_x P_n Q_{m'} f \rangle \\
&\quad = \langle WB_y^* E Q_{m'} \sqrt{f} \rangle \\
&m=0, \ldots, N_x-1, \qquad m'=0, \ldots, N_y-1
\end{aligned} \tag{9}$$

and the coefficients c_n and d_n are determined by matrix inversion.

The multiple squared coherency r^2 is the ratio of the predicted and the measured telluric field

$$r^2 = [\langle |E|^2\, W \rangle]^{-1} \left\langle \left| \sum_{n=0}^{N_x-1} c_n B_{x,n} + \sum_{n=0}^{N_y-1} d_n B_{y,n} \right|^2 W \right\rangle \tag{10}$$

It is assumed that B_x and B_y are not correlated except for some periodicities at f_j where $W(f_j)$ is set to zero. Then the right terms on the left side of Eq. (9) can be neglected and, using the orthogonality condition in Eq. (7), a quadratic equation for the errors Δc_n and Δd_n can be written (Jenkins and Watts, 1968):

$$\begin{aligned}
&\sum_{n=0}^{N_x-1} \langle |B_{x,n}|^2\, W \rangle |\Delta c_n|^2 + \sum_{n=0}^{N_y-1} \langle |B_{y,n}|^2\, W \rangle |\Delta d_n|^2 \\
&\quad \leqq \frac{\nu_1}{\nu_2} (1-r^2) \langle |E|^2\, W \rangle F_{\nu_1,\nu_2}(\beta)
\end{aligned} \tag{11}$$

with $\nu_1 = 2(N_x+N_y)$, $\nu_2 = 2(M-N_x-N_y)$; M is the number of spectral lines averaged in $\langle\ \rangle$; r^2 is the multiple squared coherency according to Eq. (10) and β is the error probability with which the value F_{ν_1,ν_2} of the Fisher-probability function is exceeded. Equation (11) describes a quadratic form with Δc_n and Δd_n as parameters which allows estimation of the maximum error of c_n and d_n for a given error probability β:

$$|\Delta c_n|^2 = \frac{\nu_1}{\nu_2} \frac{\langle |E|^2\, W \rangle}{\langle |B_{n,x}|^2\, W \rangle} (1-r^2)\, F_{\nu_1,\nu_2}(\beta)$$

$$|\Delta d_n|^2 = \frac{v_1}{v_2} \frac{\langle |E|^2 W \rangle}{\langle |B_{n,y}|^2 W \rangle} (1-r^2) F_{v_1,v_2}(\beta) \tag{12}$$

Equation (12) can be extended for related magnetic field components B_x and B_y by considering terms with $\Delta c_n \Delta d_{n'}$ in Eq.(11).

Finally, the frequency-dependent error limits $\Delta X(f)$ and $\Delta Y(f)$ of the transfer functions $X(f)$ and $Y(f)$ are calculated from Eqs. (8) and (12)

$$|\Delta X(f_i)|^2 = f_i \sum_{n=0}^{N_x-1} |\Delta c_n|^2 |P_n(f_i)|^2,$$

$$|\Delta Y(f_i)|^2 = f_i \sum_{n=0}^{N_y-1} |\Delta d_n|^2 |Q_n(f_i)|^2 \tag{13}$$

To get an idea about the optimal numbers N_x and N_y, the squared coherency r^2 from Eq.(10) is split up into the contributions $(r_{x,n})^2$ and $(r_{y,n})^2$ of each polynomial P_n and Q_n using the orthogonality condition of Eq.(7) and the assumption that B_x and B_y are uncorrelated:

$$r^2 = \sum_{n=0}^{N_x-1} \left(\frac{\langle |B_{x,n}|^2 W \rangle |c_n|^2}{\langle |E|^2 W \rangle} \right) + \sum_{n=0}^{N_y-1} \left(\frac{\langle |B_{y,n}|^2 W \rangle |d_n|^2}{\langle |E|^2 W \rangle} \right)$$
$$= \sum_{n=0}^{N_x-1} (r_{x,n})^2 + \sum_{n=0}^{N_y-1} (r_{y,n})^2 \tag{14}$$

Following an estimation of Goodman (1957), only those polynomials P_n and Q_n are used which give a significant contribution $(r_{x,n})^2$ or $(r_{y,n})^2$, e.g.

$$\left.\begin{matrix}(r_{x,n})^2 \\ (r_{y,n})^2\end{matrix}\right\} \geq \frac{10}{v_2} \tag{15}$$

with v_2 being the number of degrees of freedom. Additional polynomials will not yield a remarkably better estimation of the transfer functions X and Y.

Numerical example

The above method is applied to a set of simultaneously recorded time series of the horizontal magnetic and telluric field variations at Göttingen. Each time series exists of hourly mean values of a continuous 100-day registration. As there are long-periodic trends in the electric field it is high-pass filtered with a cut-off period of 2 days; therefore, the analysis covers the frequency range from 0.5 cpd(=cycles per day) to the Nyquist frequency of 12 cpd with frequency spacing of 0.01 cpd.

As the harmonic coeffients of B_x and B_y are very large and highly correlated in a small frequency band of 0.03 cpd width at the frequencies of the solar daily variation 1, 2, 3, 4 cpd and thus would cause the bivariate approach to fail, the weight function W is set to zero for these frequencies and to unity otherwise. It is also set to zero in a 0.03-cpd-wide frequency band centred at 1.93 cpd, the lunar tidal period $M2$, where a sharp peak in the residual electric field is found. With allowance for the missing frequencies, the number of analysed harmonic coefficients is $M = 1151 - 5 \cdot 3 = 1136$.

The squared coherency $(r_{xy})^2$ between B_x and B_y is very small for the analysed frequency interval: $(r_{xy})^2 = 0.0003$.

Table 1. Parts $(r_{x,n})^2$ and $(r_{y,n})^2$ of the estimated squared coherency r^2 according to Eq. (14). The boldface numbers exceed the significant boundary value of 0.0044 according to Eq. (15)

n	0	1	2	3	4
$(r_{x,n})^2$	**0.1405**	**0.0065**	0.0001	0.0035	0.0008
$(r_{y,n})^2$	**0.2731**	**0.0372**	**0.0162**	0.0018	0.0019

Table 1 lists the different contributions $(r_{x,n})^2$ and $(r_{y,n})^2$ to the estimated multiple squared coherency r^2.

The coherency condition according to Eq.(15) is 0.0044. It follows that $X(f)$ is sufficiently presented by the first two polynomials and $Y(f)$ by the first three polynomials.

Figure 1 shows the resulting real and imaginary parts of X and Y for N_x, $N_y = 1, \ldots, 5$ in comparison with the band-averaged estimates. The solid curves represent the transfer functions which are chosen by the coherency condition. Especially for higher frequencies do the band-averaged estimates of X scatter strongly. Consequently, additional polynomials of higher order do not alter the pattern of the transfer function. There is a significant difference just between the curves of the imaginary part of X corresponding to $N_x = 1$ and $N_x = 2$. This difference is responsible for the improved squared coherency in Table 1 and the acceptance of $N_x = 2$ as the optimal number.

There is less scattering for the estimates of Y. For the real and imaginary parts, polynomials of higher order than $N_y = 2$ do not significantly change the shape of the transfer functions. The comparison of the smooth polynomial presentation with the band-averaged transfer function shows good agreement and thus confirms the validity of the coherency condition. The following section will demonstrate that the calculation of the confidence limits is also consistent for both methods.

Treatment of disturbed electric field records

Generally, long records of electric field variations are partially interrupted because of artifical noise, problems with the electronic devices, etc., while the records of the magnetic field variations usually are less disturbed. In the case of an undisturbed record of the magnetic field, an iteration procedure is applied to estimate the transfer function on the basis of all the available electric field data.

The disturbed intervals in the electric field record are linearly interpolated and preliminary transfer functions are estimated from the *complete* time interval. The preliminary transfer functions serve to estimate the electric field variations during the gap interval. Then the calculation of the transfer functions is repeated using the new electric field record. If the ratio q of the sum of the gaps to the length of the time series is not too large, e.g. $q \leqq 0.2$, and the quality of the data is not too bad, the new estimated transfer function will yield an improved squared coherency r^2. The iteration procedure will be repeated until r^2 does not change by more than 5%.

As the iteration procedure will pretend a higher confidence of the estimation, three corrections have to be considered for the calculation of the confidence intervals. In the following and in Appendix B, the prime marks the uncorrected quantities.

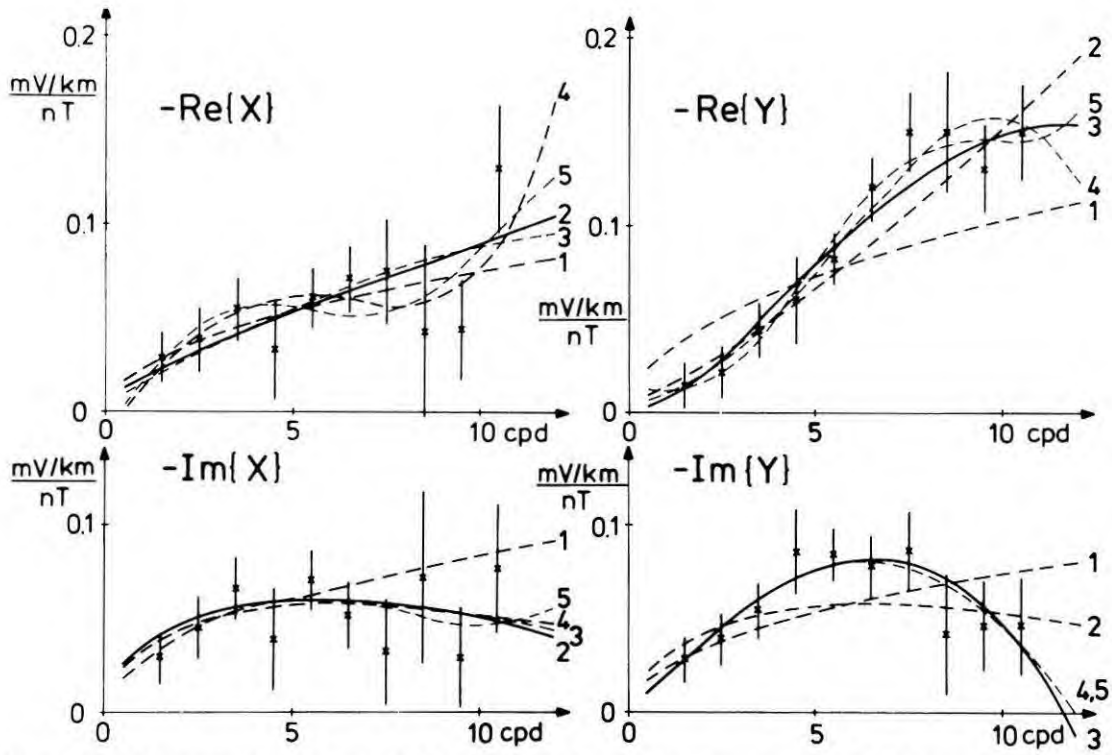

Fig. 1. Transfer functions $X(f)$ and $Y(f)$ between the telluric east component and the magnetic north and east components near Göttingen calculated from a 100-day record. *Dashed* and *solid curves* represent the polynomial representation for N_x, $N_y = 1, \ldots, 5$. *Crosses* mark the estimates which result from the band-average method (Parzen windows of equivalent bandwidth 1.0 cpd, centred at 1.5, 2.5, ..., 10.5 cpd) with 68% error bars. Confidence limits for the polynomials were omitted for clarity; they are shown in Fig. 2

a) $\nu' = 2(M - N_x - N_y)$ is the number of degrees of freedom from the whole time series as if there were no gaps. It follows directly that

$$\nu = \nu'(1-q) \tag{16}$$

b) As the power of the residual electric field $\langle|\delta E|^2\rangle$ results from the undisturbed part of the time series it has to be enlarged by the factor $1/(1-q)$

$$\langle|\delta E|^2\rangle = \frac{\langle|\delta E'|^2\rangle}{1-q} \tag{17}$$

c) The multiple squared coherency r^2 will change (see Appendix B) according to

$$r^2 = \frac{(1-q)(r')^2}{1-q(r')^2} \tag{18}$$

Figure 2 demonstrates the influence of data gaps on the estimation of the complex transfer function Y treated in the last section and its confidence intervals. For this purpose, four intervals (46, 65, 40 and 87 h) of the 100-day time series were arbitrarily chosen as gaps, thus yielding a total amount of 10% "disturbed" data. Analogously, doubled lengths of the gaps produced 20% loss of data. The transfer functions are again compared to the band-averaged estimates (see Fig. 1). The relative error of the polynomial presentation decreases towards higher frequencies, until above 10 cpd the polynomials of higher order obviously create the increase of the relative error. By the influence of data gaps, the imaginary part of Y is apparently underestimated in the frequency range around 6 cpd. In this frequency range the relative error also increases significantly. However, the polynomial approach seems to be rather stable with allowance to the confidence region.

Additionally, the size of the confidence regions calculated by the polynomial approach reflects very well the pattern of the band-averaged estimates and the size of their error bars. This fact confirms the consistency among the estimated confidence regions!

Appendix A

Calculation of the coefficients of weighted orthogonal polynomials

In Eq. (6), polynomials $P_n(f_i)$ and $Q_n(f_i)$ are introduced which fulfil the orthogonality condition in Eq. (7). The algorithm to calculate the polynomial coefficients $p_{n,l}$ *and* $q_{n,l}$ is described by Forsythe (1957) and shall be summarized for $P_n(f_i)$ only, as the calculation follows the same scheme for $Q_n(f_i)$.

Equation (6) is rewritten as

$$P_{ni} = P_n(f_i) = \sum_{l=0}^{n} p_{n,l}(f_i)^l$$
$$n = 0, \ldots, N_x, \quad i = 1, \ldots, M \tag{19}$$

Equation (7) yields

$$\sum_{i=1}^{M} P_{ni} P_{n'i} w_i = \begin{cases} \sum_{i=1}^{M} (P_{ni})^2 w_i & \text{for } n = n' \\ 0 & \text{for } n \neq n' \end{cases} \tag{20}$$

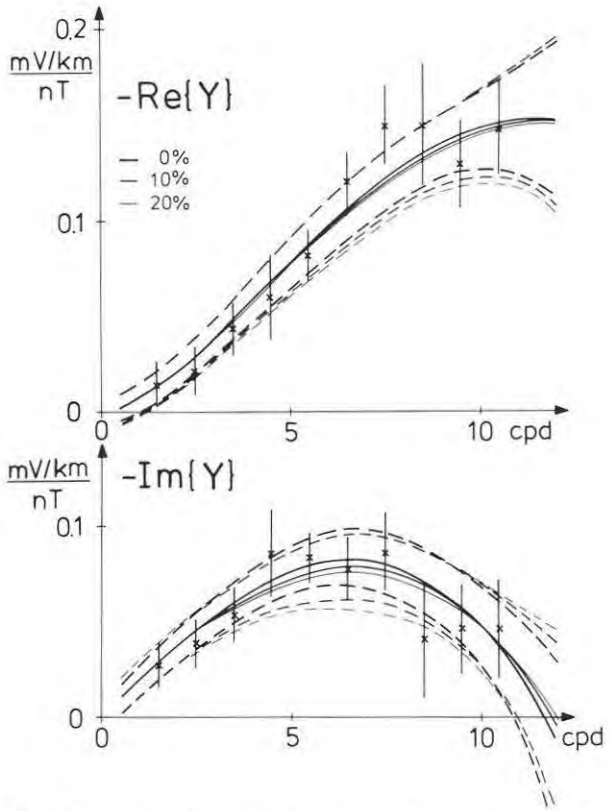

Fig. 2. Influence of disturbed data on the estimation of the transfer function Y at Göttingen. The *solid curves* refer to the polynomial approach with three polynomials including their 68% confidence limits (*dashed lines*). The decreasing thicknesses of the lines mark the estimates of the analysis with 0%, 10% and 20% disturbed data. They are compared to the band-averaged estimates (*crosses with vertical bars*) taken from the undisturbed time series as in Fig. 1

with $w_i = f_i |B_x(f_i)|^2 \, W(f_i)$ being a non-negative weight function.

The polynomials P_n are presented by a recursive formula with an additionally defined polynomial P_{-1}

$$P_{(-1)i} = 0$$

$$P_{0i} = 1$$

$$P_{(n+1)i} = (f_i - \alpha_{n+1}) \, P_{ni} - \beta_n \, P_{(n-1)i}, \qquad n = 0, \ldots, N_x - 1; \tag{21}$$

α_n and β_n can be determined using the orthogonality condition (20). Multiplying the third equation of Eq. (21) with P_{ni} and $P_{(n-1)i}$, respectively, and summing over i with allowance for w_i yields

$$\alpha_{n+1} = \frac{\sum_{i=1}^{M} f_i (P_{ni})^2 \, w_i}{\sum_{i=1}^{M} (P_{ni})^2 \, w_i} \qquad \beta_n = \frac{\sum_{i=1}^{M} f_i \, P_{ni} \, P_{(n-1)i} \, w_i}{\sum_{i=1}^{M} (P_{(n-1)i})^2 \, w_i} \tag{22}$$

The polynomial coefficients $p_{n,l}$ can be calculated using Eqs. (19) and (21). The numerical evaluation of $p_{n,l}$, however, is of minor interest, as the values of $P_n(f_i)$ are numerically determined by the recursion formula (21).

Appendix B

Squared coherency in the case of an analysis with data gaps in the telluric field

According to Eq. (1), each harmonic coefficient of the telluric field $E(f_i)$ is the sum of the predicted telluric field $E_p(f_i)$ and the uncorrelated residuum $\delta E(f_i)$

$$E(f_i) = E_p(f_i) + \delta(E(f_i)) \tag{23}$$

As $\langle \delta E' E_p^* \rangle = 0$ because of the normal equations (9), the squared coherency $(r')^2$ from Eq. (10) can be rewritten as

$$r^2 = \frac{\langle |E_p|^2 \rangle}{\langle |E|^2 \rangle} = \frac{1}{1 + \langle |\delta E|^2 \rangle / \langle |E_p|^2 \rangle} \tag{24}$$

Let q be the ratio of the sum of the intervals with missing data to the length of the time series. Then the calculated residuum $\langle |\delta E'|^2 \rangle$ is, according to Eq. (17),

$$\langle |\delta E'|^2 \rangle = (1 - q) \langle |\delta E|^2 \rangle \tag{25}$$

Therefore, the calculated coherency $(r')^2$ is biased and has to be corrected by inserting Eq. (25) into Eq. (24)

$$r^2 = \frac{1}{1 + \langle |\delta E'|^2 \rangle / [(1-q) \langle |E_p|^2 \rangle]} = \frac{(1-q)(r')^2}{1 - q(r')^2} \tag{26}$$

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References

Forsythe, G.E.: Generation and use of orthogonal polynomials for data-fitting with a digital computer. J. Soc. Indust. Appl. Math. **5**, No. 2, 74–88, 1957

Goodman, N.R.: On the joint estimation of the spectra, co-spectrum and quadrature spectrum of a two-dimensional stationary Gaussian process. Scientific paper **10**, Engineering Statistics Lab., New York University 1957

Jenkins, G., Watts, P.: Spectral analysis and its applications. San Francisco: Holden Day 1968

Larsen, J.C.: Low frequency (0.1–6.0 cpd) electromagnetic study of deep mantle electric conductivity beneath the Hawaiian Islands. Geophys. J. R. Astron. Soc. **43**, 17–46, 1975

Larsen, J.C.: Electromagnetic response functions from interrupted and noisy data. J. Geomagn. Geoelectr. **32**, 17–46, 1980

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